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P-Parity Violating Bound States of Particles with Anomalous Magnetic Moment[1]

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Abstract

We consider bound states of fermions with an anomalous magnetic moments(neutrinos,neutrons) in radial electric and magnetic field of monopole.In case of the radial magnetic field the interaction $\vec{\Sigma}\vec{H}$ violates P-parity and for this reason we must use the method of [10](hep-ph/9901248) where both components of spinor considererd as a linear combination of spheric spinors with different P-parity.Also we apply pseudoscalar-like interaction (2) obtained in [3] to monopole case and add it in Dirac equation.We obtain the system of differential equations for radial functions which define energy levels of fermions with anomalous magnetic moments in the presence of monopole.

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As known, the dynamics of the neutral fermions with anomalous magnetic moments are described by Dirac equation with nonminiaml coupilngs of neutral fermions with electromagnetic field [1],[2]:

$$(\hat{k} - m_n + \mu_n(\vec{\Sigma}\vec{B} - i\vec{\alpha}\vec{E}) + iq(\vec{E}\vec{B})\gamma_5)\psi(k) = 0, \tag{1}$$

where μ_n-is anomalous magnetic moment, $\frac{1}{2}\vec{\Sigma}$ is spin operator,and defined by formula (21,21) of [1] operator $\vec{\alpha}$ is defined by formula (21,20) of [1].

In this equation we include also term obtained in ref. [3]:

$$L = iq(\vec{E}\vec{H})\bar{\psi}\gamma_5\psi \tag{2}$$

which appears e.g. in electroweak models at one-loop level in theories with P-parity violation(for electroweak thories and P-parity violation see e.g. [11] and references therein).

In this article we consider bound states of particles with anomalous magnetic moments and with interaction (2) in the presence of monopole (see [4] [5] references in [6]). We obtained generalization of equations (13),(14) [9] where has been considered bound states of particles with anomalous magnetic moments in arbitrary radial electric field.

In [9] has been considered joint influence of the static radial electric field and magnetic fild.Although in this paper B is not radial as has been shown that in some condition term $\vec{\Sigma}\vec{B}$ is radial.However in contrast to monopole case in [9] term $\vec{\Sigma}\vec{B}$ is P-parity conserved.

The magnetic field (but not magnetic field of monopole) has been presented in equations (13),(14) of ref. [9] besides radial electric field. During derivation of equations (12)-(15) below which defines energy levels of the fermions with anomalous magnetic moment in the presence of monopole the

method of the [10] has been used (because in case of radial magnetic field the term $\vec{\Sigma}\vec{B} \sim \vec{\Sigma}\vec{r}$ violates P-parity) for angular variables separation, in accordance with this method it is necessary to present components of spinors as linear combination of spheric spinors $\Omega_{jlM}(\vec{n}), \Omega_{jl'M}(\vec{n})$ which have different P-parity. We also confirm the result of [9] where has been stressed that in case of Coulomb electric field take place the fall down on the center takes place.

As known(see (see e.g. references in [6])) exist nontrivial solutions of to Yang-Mills theories which have in general both electric and magnetic fields:

$$\vec{B} = \vec{n}B(r) \tag{3}$$

$$\vec{E} = \vec{n}E(r) \tag{4}$$

where$\vec{n} = \frac{\vec{r}}{r}$.At large distances from the core of the monopole we have:

$$\vec{E} = \frac{e\vec{r}}{r^3} \tag{5}$$

$$\vec{B} = \frac{g\vec{r}}{r^3} \tag{6}$$

It must be noted that between between electric and magnetic charges there exist a relation like that between electric and magnetic charges in case of Dirac monopole [7] :

$$eg = \frac{1}{2}n, \quad n = 0, \pm 1, \pm 2, \pm 3, ... \tag{7}$$

Thus below we consider bound states of neutral fermions with anomalous magnetic moment in both radial magnetic and electric fields created by these objects.

Also it is of interest to note that in case of monopole $\vec{E}\vec{B} = E(r)B(r)$ and thus pseudoscalar interaction (2) is spherically symmetric (depends only on

r) as well as the interaction connected with anomalous magnetic moment and separation of angular variable is possible as seen below.At large distances:

$$\vec{E}\vec{B} = E(r)B(r) = eg\frac{1}{r^4} \tag{8}$$

and thus we have radial interaction (2) which is attractive at appropriate sign of n and lead to fall down on the center.The non-locality of $L = iq(\vec{E}\vec{B})\bar{\psi}\gamma_5\psi$ vertex and finite size of monopole prevent this fall down.Besides, the term:

$$q^2(\vec{E}\vec{B})^2 = q^2\frac{e^2g^2}{r^8} \tag{9}$$

in effective potential is always repulsive and dominates at small distances.

It must be stressed that $\vec{E}\vec{B}$ depends only on r because $\vec{B}$ is radial. Also radial is the interaction of particles with anomalous magnetic moment with electric and magnetic fields of monopole in Dirac equation for particles with anomalous magnetic moment :

$$(\hat{k} - M(r) + \mu(g\vec{\Sigma}\vec{n}B(r) - ie\vec{\alpha}\vec{n}E(r)) + iqE(r)B(r)\gamma_5)\psi(k) = 0, \tag{10}$$

Here $M(r) = c\phi(r)$ ($\phi(r)$- is Higgs field which give mass to the fermion, $m = M(\infty)$-is the observed mass of fermion).

Here P-violation presented (because interaction (2) is effectively pseudoscalar, $\vec{\Sigma}\vec{n}B(r)$ also P-odd)and we find as in [10] the solution as linear combinations of spheric spinors $\Omega_{jlM}(\vec{n}), \Omega_{jl'M}(\vec{n})$ which have different P-parity:

$$\begin{aligned}
&\psi^T = (\phi, \chi), \\
&\phi = f_1(r)\Omega_{jlM}(\vec{n}) + (-1)^{\frac{1+l-l'}{2}} f_2(r)\Omega_{jl'M}(\vec{n}), \\
&\chi = g_1(r)\Omega_{jlM}(\vec{n}) + (-1)^{\frac{1+l-l'}{2}} g_2(r)\Omega_{jl'M}(\vec{n}))
\end{aligned} \tag{11}$$

After separation of angular variables we obtain the following set of equations for radial functions:

$$f_1'(r)+\frac{1+\kappa}{r}f_1(r)+\mu E(r)f_1(r) = (\epsilon+M(r))g_1(r)-i\mu B(r)g_2(r)+a_P(r)f_2(r) = 0 \tag{12}$$

$$f_2'(r)+\frac{1-\kappa}{r}f_2(r)+\mu E(r)f_2(r) = -(\epsilon+M(r))g_2(r)-i\mu B(r)g_1(r)-a_P f_1(r) = 0 \tag{13}$$

$$g_1'(r)+\frac{1-\kappa}{r}g_1(r)-\mu E(r)g_1(r) = -(\epsilon-M(r))f_1(r)+i\mu B(r)f_2(r)+a_P g_2(r) = 0 \tag{14}$$

$$g_2'(r)+\frac{1+\kappa}{r}g_2(r)-\mu E(r)g_2(r) = (\epsilon-M(r))f_2(r)+i\mu B(r)f_1(r)-a_P g_1(r) = 0 \tag{15}$$

where

$$\kappa = l(l+1) - j(j+1) - \frac{1}{4}, \tag{16}$$

$$a_P = iqE(r)B(r), \tag{17}$$

At large distances $a_P = \frac{qeg}{2r^4}$.

In pure electric field case i.e. at $B = 0, q = 0$ P-parity is conserved and we obtain two decoupled system of equations (13),(14) of the [9] in which of course the magnetic field is also zero.

If only electric field is presented we obtain for radial functions the following equations:

$$(\vec{p}^2 + m^2 - \epsilon^2 + \mu^2 E^2 + 4\mu\pi\rho \pm \frac{2\mu E(r)(1+\kappa)}{r})R_{1,2}(r) = 0. \tag{18}$$

where $\vec{p}^2 = -\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}+\frac{l(l+1)}{r^2}$ (we find the solution as $\phi = R_1(r)\Omega_{jlM}(\vec{n}), \chi = R_2(r)\Omega_{jl'M}(\vec{n})$). During derivation of this formulas we take into account that $div\vec{E} = 4\pi\rho$.

In [9] it has been stressed (page 3 after formulas (13),(14)) that for particles with anomalous magnetic moment we have fall down on the center in case of Coulomb electric field which prevented by cut off of the potential, i.e. by taking into account charge distribution inside neutron.

Indeed, from equations (18)-(19)(as well as from equations (13)(14) of the ref.[9]) it is seen that in case of Coulomb attraction ($E(r) = \frac{Ze}{r}$) we have fall down on the center due to the term $\mu \frac{E(r)}{r}(1+\kappa) = \frac{\mu e Z(1+\kappa)}{r^3}$in effective potential. It must be noted, however, that the term $\mu^2 E^2 = \frac{Z^2 \mu^2 e^2}{r^4}$ in potential is always repulsive and at small r prevents fall down on the center. In case of pointlike charge term $2\pi\rho = 2\pi\mu Z e \delta(\vec{r})$ may be considered as perturbation.

In case of Coulomb potential we can calculate by using quasiclassical methods energy levels which are defines by the equation [8]:

$$\int \sqrt{2m(E - V(r)} dr = n \tag{19}$$

where

$$V(r) = \frac{(l+\frac{1}{2})^2}{2mr^2} + \frac{\mu E(1+\kappa)}{r} + \frac{1}{4}\mu^2 E^2 \tag{20}$$

$$2mE = \epsilon^2 - m^2 \tag{21}$$

Appendix

Below we present system of equation for radial functions (19)-(22) of ref. [10] which obtained after angular variables separation in electroweak Dirac equation:

$$(f_1'(r) + \frac{1+\kappa}{r} f_1(r)) - (E + M(r) - V(r)) g_1(r) - V_+(r) f_2(r) = 0 \tag{22}$$

$$(f_2'(r) + \frac{1-\kappa}{r} f_2(r)) + (E + M(r) - V(r))g_2(r) + V_+(r)f_1(r) = 0 \quad (23)$$

$$(g_1'(r) + \frac{1-\kappa}{r} g_1(r)) + (E - M(r) - V(r))f_1(r) + V_-(r)g_2(r) = 0 \quad (24)$$

$$(g_2'(r) + \frac{1+\kappa}{r} g_2(r)) - (E - M(r) - V(r))f_2(r) - V_-(r)g_1(r) = 0 \quad (25)$$

where:

$$M(r) = m - a_S V_S(r) \quad (26)$$

$$V(r) = eg_V Z_0(r) + eQA_0(r), \quad (27)$$

$$V_\pm(r) = eg_A Z_0(r) \pm a_P V_P(r), \quad (28)$$

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